AN ALTERNATIVE WAY TO OPTIMIZING MESSAGING DISTRIBUTION APPLYING CHINESE POSTMAN ALGORITHM (CPP) TESTING WITH COMPLEMENTARY SLACKNESS THEOREM USING MULTIPLE PATHS. INSIDE MEXICAN PUBLIC INSTITUTION

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Abstract

This paper presents an efficient way to manage the distribution of messaging in a public company in Mexico, modeling multiple route options with the objective to minimize the delivery time as well as the use of resources to achieve to meet the target. The work is based on the algorithm of the Chinese Postman uses linear programming as a way to reach the optimization model. The way to build it is simple and practical, it may be useful to private and public companies, obtaining and interpretation of results are produced through a very effective code to run in polynomial time with amazing times and translation solution is relatively simple and the implementation costs will result very competitive.. In this time of globalization, enterprises and institutions must take refuge in science as a way of making smart, rational choices. Analogously complementary slackness theorem is applied to show the power of the theorem in order to guarantee the optimization of a linear model with multicapacity. In an alternate mathematical tool .

Key Words. Postman, Slackness, odd flow,node.

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Objective.

This research document show an interesting algorithm for undertaking that distribute products or services across pipeline, messaging routes or another using multiple branches and how it can be minimizing distances and associated factors such as time and money, when you need to travel at least twice on the same route. By other hand the solution is tested using marvelous tool called the complementary slackness theorem how alternative to raise the competitiveness required in world class companies.

Introduction.

In 1962themathematician namedKuanMei-Ko wasinterested in howpostal staffcoulddeliver the lettersinto a numberofblockssuch thatthetotal distance walkedby themailmanis asshortas possible.

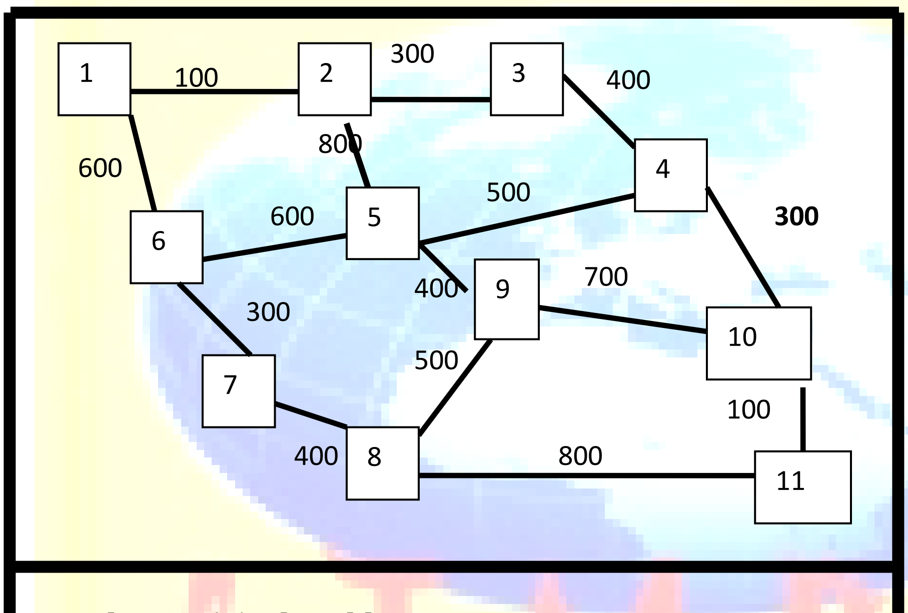
TheCPPis one of theclassical problemsindiscrete mathematicsalsois anequivalent tothe travelagent (TSP) problemisposedas follows: A portfolioorpostmanshouldgo anumber ofcity streets, visit eachat least onceanddeliver mailand then returnto theirhomeoffice(Eulerian cycle), Jarvis(2005), is tofind a routetothe postmanthat minimizesthetotal distance traveled. Ifthe networkhas anEulerian cycleis clearthat thiswillsolve theproblem anotherway, surely some vertexisofodddegreewhichwill leadtours oftheedge(s) more than once. Recall thatan Euleriancycle.It isthiscycle thatpasses through all thenodesonceand traverseseach edgeexactly and strictly only once, otherwise you will need to convert the original problem across Eulerian cycle

The stepsto resolvethis problemis summarized inthe followingparagraphs,Barnes(2005):

Step 1	Identifyingflownodesoddconsideringthe arcsthat are related tothem.
Step 2	These nodesmust appearconsideringall possible pairs.
Step 3	Findingthe shortest distance betweeneach pair ofoddnodes.
Step 4	Choose the set of odd nodes including node and select a pair that route with minimum total distance. The peer node is added to the sum of all the arcs between odd nodes to give the smallest distance to traverse the network and return to the original point.

ImplementationProposal

Solve thefollowing problemshownin Graph 1 oncourier deliveryofthe MigrantMinistryof the State Government in Mexicoat differentworkplacesusing thePCCalgorithm tothe following offices, the lines representrelationships betweenagencies andshownbesidesa number thatis thelength thereof, it can be seenthat the position ofthe post office(source)does not affectthe solution.



Graph. 1 Original Problem.

Step 1

Node Number	Number of Edges	Type Node	Node Number	Number of Edges	Type Node
6	3	**Odd**	1	2	**Pair**
9	3	**Odd**	3	2	**Pair**
4	3	**Odd**	5	4	**Pair**
2	3	**Odd**	7	2	**Pair**
8	3	**Odd**	11	2	**Pair**
10	3	**Odd**			

Table I.

Table of Pairs and Odd Nodes in the Network.

Step 2

Nodes and Names	Relation between Work Centers	Distances in meters
1. Secretary of Migrant	1-2	100
2. Administration Bureau	1-6	600
3. Particular Secretary	2-3	300
4. SocialService Bureau	2-5	800
5. Human Resources Bureau	3-4	400
6. Informatics Department	4-5	500
7.Benefits Office	4-10	300
8. Training Bureau	5-6	600
9. Legacy Office	5-9	400
10.Audience Bureau	6-7	300
11 Workshops Studio	7-8	400
12.Ofice Files	8-9	500
13. Social Communication Bureau	8-11	800
14.Pay Rolls and Salaries Bureau	9-10	700
15.Labor Relations Bureau	10-11	100

Table II

Step 3

Graph odd nodes is done by calculating the minimum distance considering all nodes scam nominal flow

(input and output) .

For example;If you want to go since node number 2 (two) even node number ten (10) The total Distance Shortest Path is as follow ;

2 ⇨3 ⇨ 4 ⇨ 10 with flow equal to 300 + 400 + 300 in summary we have total flow of 1000 . You can see how we can go directly since node number two (2) even node number ten(10) in the network.
2⇨ 10 total flow equal to 1000

This leads makes a simplification of nodes , as such in the graph note on the third node (3) only has one input stream and an output stream having a total time and considers two flows (pair) , which is adds the value of these flows and disappears from the network analysis. Graph 2.

Step 3

Chinese Postman Algorithm to detail

Odd Node	Matrix of shortest distances between odds nodes				
	4	**6**	**8**	**9**	**10**
2	300+400=700	100+600=700	100+600+300+400=1400	800+400=1200	300+400+300=1000
4		500+600=1100	300+100+800=1200	500+400=900	300
6			300+400=700	600+400=1000	600+500+300=1400
8				500	800+100=900
9					700
10					

Table III

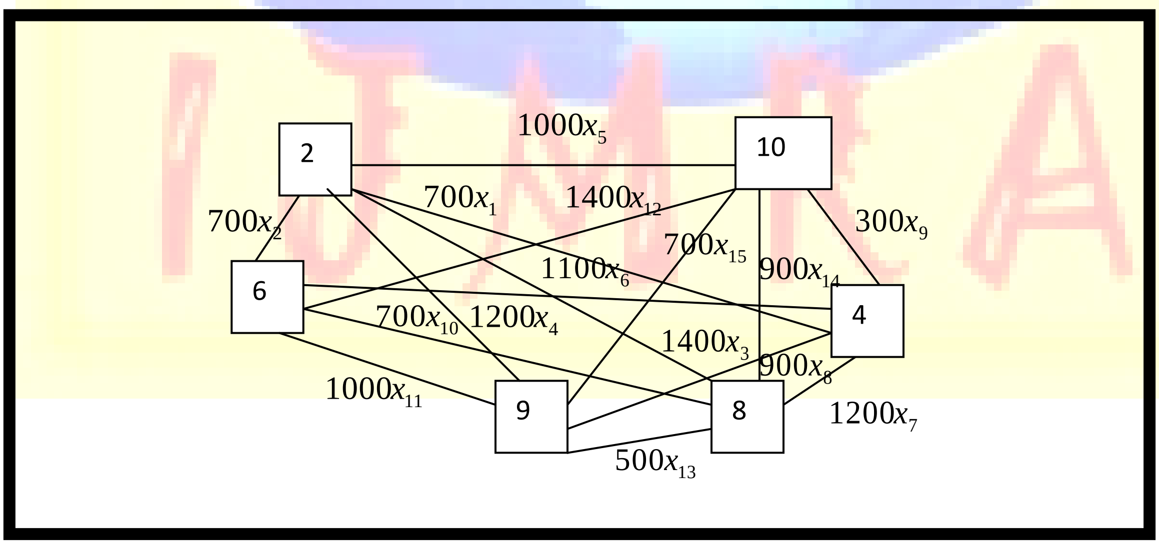
Step 4

Chinese Postman Algorithm to detail

Odd Node	Variables in the network graphic between odds nodes.				
	4	**6**	**8**	**9**	**10**
2	X1	X2	X3	X4	X5
4		X6	X7	X8	X9
6			X10	X11	X12
8				X13	X14
9					X15
10					

Table IV

Step 4



Graph 2

Once computed the minimum distances only between nodes with odd line flow you can see the graph 2, Note that now all network nodes have odd degree, which will result in more than one tour at some edges, Therefore integrated into the objective function and generating constraints for each odd node to be codified model and subsequent analysis on the results produced by the software.

Note that nodes 1, 3, 5, 7 and 11 are considered in the network of graph number II, but they do not appear as such, we are now able to set the linear programming model based on the above graph.

The linear programming model is as follows;

$$Min, 700x_1 + 700x_2 + 1400x_3 + 1200x_4 + 1000x_5 + 1100x_6 + 1200x_7 + 900x_8 + 300x_9 + \ldots\ldots + 700x_{10} + 1000x_{11} + 1400x_{12} + 500x_{13} + 900x_{14} + 700x_{15}$$

$$subject, to$$

$$x_1 + x_2 + x_3 + x_4 + x_5 <= 1, (node.2)$$

$$x_1 + x_6 + x_7 + x_8 + x_9 <= 1, (node.4)$$

$$x_2 + x_6 + x_{10} + x_{11} + x_{12} <= 1, (node, 6)$$

$$x_3 + x_7 + x_{10} + x_{13} + x_{14} <= 1, (node.8)$$

$$x_4 + x_8 + x_{11} + x_{13} + x_{15} <= 1, (node.9)$$

$$x_5 + x_9 + x_{12} + x_{14} + x_{15} <= 1, (node.10)$$

Table V

Programming Linear Code
*** Chinese Postman Algorithm**
***Performance by Dr. Francisco Zaragoza**
Sets
j / 1*15 /
i / 1*6/ ;
Parameters
B(i) / 1 1
2 1

```
        3      1
        4      1
        5      1
        6      1/;
Parameters
C(j) /  1      700
        2      700
        3      1400
        4      1200
        5      1000
        6      1100
        7      1200
        8      900
        9      300
        10     700
        11     1000
        12     1400
        13     500
        14     900
        15     700   /;
Variables
X(j),z
BINARY  variables
X(j)  ;
table A(i,j)
1     2     3     4     5     6     7     8     9     10     11     12     13     14
15
1     1     1     1     1     1
2     1                       1     1     1     1
```

3	1				1				1	1	1			
4		1				1			1			1	1	
5			1				1			1		1		1
6				1				1			1		1	1

Equations

fo Objective Function

rest(i) Constraint;

fo .. z =e= sum (j, c(j)*x(j));

rest(i).. sum (j,A(i,j)*x(j))=E= b(i);

Model Exercise /all/;

Solve Exercise using MIP Minimizing z;

Table VI

Experimental Results

MODEL STATISTICS

BLOCKS OF EQUATIONS	**2**	**SINGLE EQUATIONS**	**7**
BLOCKS OF VARIABLES	**2**	**SINGLE VARIABLES**	**16**
NON ZERO ELEMENTS	**46**	**DISCRETE VARIABLES**	**15**

GENERATION TIME = 0.047 SECONDS **4 Mb WIN237-237 Aug 23, 2011**

EXECUTION TIME = 0.047 SECONDS

S O L V E S U M M A R Y

MODEL ejerc OBJECTIVE z
TYPE MIP DIRECTION MINIMIZE
SOLVER CPLEX FROM LINE 50
****** SOLVER STATUS 1 Normal Completion**
****** MODEL STATUS 1 Optimal**
****** OBJECTIVE VALUE 1500.0000**

Table VII.

- VAR x

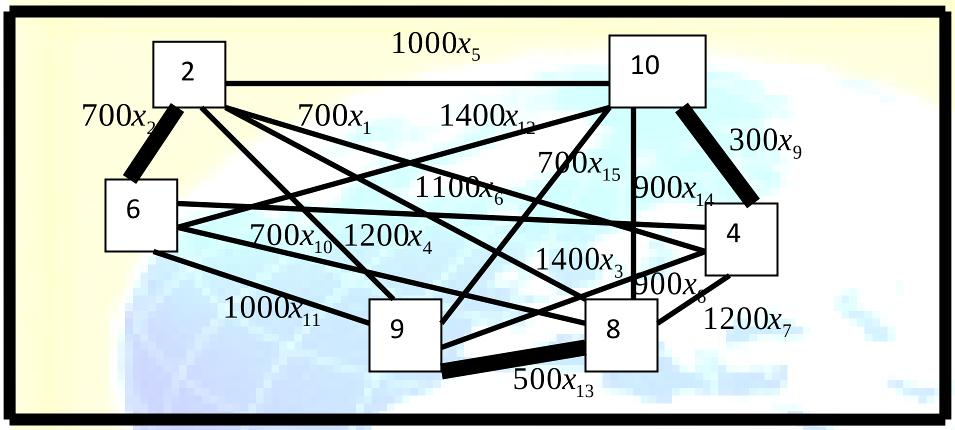
	LOWER	LEVEL	UPPER	MARGINAL
1	**.**	**.**	**1.000**	**700.000**
2	**.**	**1.000**	**1.000**	**700.000**
3	**.**	**.**	**1.000**	**1400.000**
4	**.**	**.**	**1.000**	**1200.000**
5	**.**	**.**	**1.000**	**1000.000**
6	**.**	**.**	**1.000**	**1100.000**
7	**.**	**.**	**1.000**	**1200.000**
8	**.**	**.**	**1.000**	**900.000**
9	**.**	**1.000**	**1.000**	**300.000**
10	**.**	**.**	**1.000**	**700.000**
11	**.**	**.**	**1.000**	**1000.000**
12	**.**	**.**	**1.000**	**1400.000**
13	**.**	**1.000**	**1.000**	**500.000**
14	**.**	**.**	**1.000**	**900.000**
15	**.**	**.**	**1.000**	**700.000**

S O L V E S U M M A R Y

Table VIII

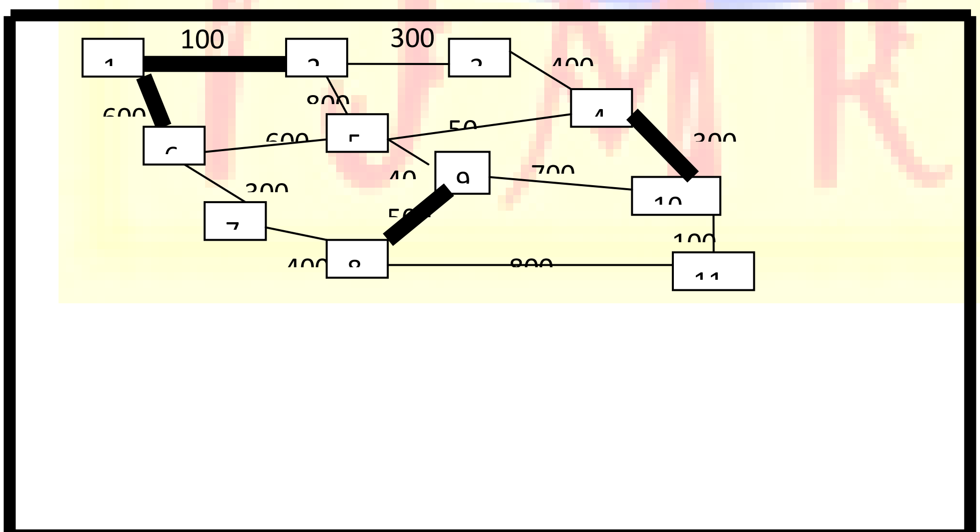
Variable x			
Coger	**Level**	**Upper**	**Marginal**
-INF	**1500.00**	INF	.

Graph3 shows the results obtained from the Gams software and integrated graphics, the minimum distances of odd nodes under study.



Graph 3

About the graph number 3 which is equivalent to the following final graph 4 where the results based on the original network are implemented.



Graph 4

From the above results and analyzing the graph 4, above, is observed that the minimum distance is equal to 1500 so it must be the routes that should be used twice for the minimum time must:

Experimental Results Path	Names Nodes
Entre el nodo 6 y 1	**Secretary of Migrant and Informatics Department**
Entre el nodo 1 y 2	**Secretary of Migrant and Administration Bureau**
Entre el nodo 4 y 10	**Social Service Bureau and Audience Bureau**
Entre el nodo 9 y 8	**Training Bureau and Legacy Office**

Table IX.

Solution Path

The staff must leave the node 1 goes through all edges at least once , through all the edges and performs dual path between nodes (6,1) , (1,2) , (4.10) and (9, 8) , so that leaves and returns to the origin.

The route for the staff starts and ends at node 1																			
1	**6**	**7**	**8**	**9**	**8**	**11**	**10**	**4**	**10**	**9**	**5**	**4**	**3**	**2**	**1**	**6**	**5**	**2**	**1**

Table X

See Graph 4

- It is very interesting to observe the execution time Only 0.047 seconds a timer job simply spectacular .
- The information generated can be translated in time for money overall efficiency for performance evaluation of existing systems and continuous improvement scenarios , is the art of mathematical programming to its fullest.

Application of the condition of Complementary Slackness Theorem. To the original problem solved by C.P.P. Algorithm

One of the main theorems in the theory of linear programming duality theorem Complementary clearance . This theorem allows us to find the optimal solution of the dual problem when we know the optimal solution of the primal problem (and reciprocally) by solving a system of equations consisting of the decision variables

(primal and dual) and restrictions (model primal and dual) .

The importance of this theorem is that it facilitates the resolution of the models of linear optimization , allowing who solves find the simplest model to address (from the algorithmic point of view) given that in any way you can get the results of the equivalent model associated (is this the model primal or dual) .

This is a very important theorem relating the primal and dual problems, which obviously indicates that at least one of the two terms in each particular expression must be zero. Bazaraa (2005)

If x* and w* are any solutions to the problem Primal and Dual problems in canonical form pair respectively .

We have to ;

Cx* = w* Aw* = w*b
But Cx* = w*b;
Therefore Cx*= w*Aw*=w*b
being obtained w* (Ax*-b) = 0 y (C-w*A) x* = 0

Table XI

Complementary Slackness Theorem.

(cj-w*aj) xj* = 0; **para** j=1,2,.....,n
w*i (ai x* - bi) = 0, **para** i =1,2,....,m

Table XII

In a linear programming problem when optimality is reached. If a variable in a problem is non-zero , then the corresponding equation in the other mutual problem must be set (Ax- b = 0) and an equation if a problem does not fit (Ax - b 0), then the corresponding variable in the other reciprocal problem must be zero.

Proposed Allocation of Public Administration

Given the optimal solution of the dual problem, the optimal solution using Chinese Postman algorithm (original problem) using Complementary Slackness Theorem.

Primal Model of Original problem

$$Min, 700x_1 + 700x_2 + 1400x_3 + 1200x_4 + 1000x_5 + 1100x_6 + 1200x_7 + 900x_8 + 300x_9 + \ldots\ldots\ldots\ldots + 700x_{10} + 1000x_{11} + 1400x_{12} + 500x_{13} + 900x_{14} + 700x_{15}$$

$$subject, to$$

$$x_1 + x_2 + x_3 + x_4 + x_5 <= 1, (node.2)$$
$$x_1 + x_6 + x_7 + x_8 + x_9 <= 1, (node.4)$$
$$x_2 + x_6 + x_{10} + x_{11} + x_{12} <= 1, (node.6)$$
$$x_3 + x_7 + x_{10} + x_{13} + x_{14} <= 1, (node.8)$$
$$x_4 + x_8 + x_{11} + x_{13} + x_{15} <= 1, (node.9)$$
$$x_5 + x_9 + x_{12} + x_{14} + x_{15} <= 1, (node.10)$$

Table XIII

Dual model to Primal Problem using CPP Algorithm

$$
\begin{aligned}
&Max, w = y1 + y2 + y3 + y4 + y5\\
&Sujeto, a\\
&y1 + y2 <= 700\\
&y1 + y3 <= 700\\
&y1 + y4 <= 1400\\
&y1 + y5 <= 1200\\
&y1 + y6 <= 1000\\
&y2 + y3 <= 1100\\
&y2 + y4 <= 1200\\
&y2 + y5 <= 900\\
&y3 + y4 <= 300\\
&y3 + y5 <= 1000\\
&y3 + y6 <= 1400\\
&y4 + y5 <= 500\\
&y4 + y6 <= 900\\
&y5 + y6 <= 700\\
&y1, y2, y3, y4, y5 >= 0
\end{aligned}
$$

Table XIV

Experimental Results.

Global optimal solution found using Lindo Software	
Objective value:	**1500.000**
Infeasibilities:	**0.000000**
Total solver iterations:	**6**
Model Class:	**LP**
Total variables:	**6**
Nonlinear variables:	**0**
Integer variables:	**0**

Total constraints:	16	
Nonlinear constraints:	0	
Total nonzeros:	35	
Nonlinear nonzeros:	0	
Variable	**Value**	**Reduced Cost**
Y1	400.0000	0.000000
Y2	300.0000	0.000000
Y3	300.0000	0.000000
Y4	0.000000	1.000000
Y5	500.0000	0.000000
Y6	0.000000	0.000000
Row	**Slack or Surplus**	**Dual Price**
1	1500.000	1.000000
2	0.000000	1.000000
3	0.000000	0.000000
4	1000.000	0.000000
5	300.0000	0.000000
6	600.0000	0.000000
7	500.0000	0.000000
8	900.0000	0.000000
9	100.0000	0.000000
10	0.000000	0.000000
11	0.000000	1.000000
12	200.0000	0.000000
13	1100.000	0.000000

14	0.000000	1.000000
15	900.0000	0.000000
16	200.0000	0.000000

Table XV

Substituting the solution into the dual problem values are;

$y1 + y2 <= 700; 400 + 300 <= 700, satisfied, equation$
$y1 + y3 <= 700; 400 + 300 <= 700, satisfied, equation$
$y1 + y4 <= 1400; 400 + 0 <= 1400, not, satisfied, equation$
$y1 + y5 <= 1200; 400 + 500 <= 1200, not, satisfied, equation$
$y1 + y6 <= 1000; 400 + 0 <= 1000, not, satisfied, equation$
$y2 + y3 <= 1100; 300 + 300 <= 1100, not, satisfied, equation$
$y2 + y4 <= 1200; 300 + 0 <= 1200, not, satisfied, equation$
$y2 + y5 <= 900; 300 + 500 <= 900, not, satisfied, equation$
$y2 + y6 <= 300; 300 + 0 <= 300, satisfied, equation$
$y3 + y4 <= 700; 300 + 0 <= 700, not, satisfied, equation$
$y3 + y5 <= 1000; 300 + 500 <= 1000, not, satisfied, equation$
$y3 + y6 <= 1400; 300 + 0 <= 1400, not, satisfied, equation$
$y4 + y5 <= 500; 0 + 500 <= 500, satisfied, equation$
$y4 + y6 <= 900; 0 + 0 <= 900, not, satisfied, equation$
$y5 + y6 <= 700; 500 + 0 <= 700; not, satisfied, equation$

Table XVI

Note: Where it is mentioned that the equation is adjusted , it means reached exactly the resource value of the right side , otherwise it is not reached. Which means:

That the values are perfectly satisfied equations **1, 2, 9 and 13**

As with the subscript variables in the primal problem will have a nonzero value and the equations that are not satisfied we will assume that the variables with the subscript number of the equation have zero value in the primal.

Consider the primal equations where x1 , x2, x9 and x13 variables that have zero value are embedded , all other variables have zero, which allows us to clear and know the value of the primal variables we seek.

$$x_1 + x_2 + x_3 + x_4 + x_5 <= 1, (node2)$$
$$x_1 + x_6 + x_7 + x_8 + x_9 <= 1, (node4)$$
$$x_2 + x_6 + x_{10} + x_{11} + x_{12} <= 1, (node6)$$
$$x_3 + x_7 + x_{10} + x_{13} + x_{14} <= 1, (node8)$$
$$x_4 + x_8 + x_{11} + x_{13} + x_{15} <= 1, (node9)$$
$$x_5 + x_9 + x_{12} + x_{14} + x_{15} <= 1, (node10)$$

Table XVII

$$Substituting, values, in, primal, problem, u\sin g, dual, problem, therefore, we, have;$$
$$x1 + x2 + 0 + 0 + 0 <= 1, (node,2)$$
$$x1 + 0 + 0 + 0 + x9 <= 1, (node,4)$$
$$x2 + 0 + 0 + 0 + 0 <= 1, (node,6)$$
$$0 + 0 + 0 + 0 + x13 + 0 <= 1, (node,8)$$
$$0 + 0 + 0 + x13 + 0 <= 1, (node,9)$$
$$0 + x9 + 0 + 0 + 0 <= 1, (node,10)$$

Table XVIII

$$Min, 700x_1 + 700x_2 + 1400x_3 + 1200x_4 + 1000x_5 + 1100x_6 + 1200x_7 + 900x_8 + 300x_9 +$$
$$+ 700x_{10} + 1000x_{11} + 1400x_{12} + 500x_{13} + 900x_{14} + 700x_{15}$$

Z*=700 (x2) + 300(x9) + 500(x13)
Z*=700(1) + 300(1)+500(1)=1500

Table XIX

Whereupon the values of the problem variables Primal and the optimal solution of the objective function by applying the complementary slackness conditions of Dual problems and can verify that the optimal value of the problem is obtained Primal and dual problem are obtained being in this case.

Z*=W*=1500

Lagging then demonstrated the power of this tool that allows linear programming to obtain the value of the objective function practically and efficiently from the dual problem with the complementary slackness theorem, a fundamental tool in the optimization processes.

Conclusions.

- We can see how is possible to minimize distances and the number of returns on the original way using an efficient algorithm.
- You can watch how is possible design a model across network using optimization with excellent results.
- The complementary slackness theorem is a great tool to test the original model.
- The economy of implementation results cheaper than another methods. Andso easy to understand the answers.
- We can show an original programming code to use large scale optimization.
- We can infer based on the above equations that the values of;

x1 = 0, x2 = 1, x9 = 1, and x13 = 1

- Interesting as x1 should apparently take zero value, however , to perform algebraic operations is worth zero. Once this detail above can obtain the value of the objective function by substituting the values of the variables and multiplication with the respective coefficient, as shown below:
- Total Distance is equal to 1500 meters.

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